FORMLABS LESSON PLAN

The Dodecahedron

An artistic exploration into links between our 3D world and 2D methods of visualization.

Adapted by Formlabs, original submission by Chris Sloan of Science Visualization.
Formlabs’ Innovate & Educate Challenge invited educators across the country to develop and share lesson plans that encourage creative thinking and hands-on learning through 3D printing. Thanks to those who submitted, we’re able to share free resources with a wider learning community dedicated to inspiring students with rich, immersive experiences.

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**SUMMARY**
The Dodecahedron makes use of a 3D printed kit to introduce students to hands-on methods of translating three-dimensional objects into two-dimensional space. The lesson draws historical connections to notable mindsets and techniques around orthographic views, linear perspective drawings, dodecahedron geometry, the golden ratio, and the Fibonacci series. Students will journey from pre-Renaissance schools of thought, through how da Vinci’s contributions have shaped how we depict our world, to innovations enabled by 3D modeling advancements.

**OBJECTIVES**
- Learn about translating three-dimensional objects into two dimensions
- Learn about Platonic solids
- Learn about geometric ratios in nature and in design
- Learn about projective geometry

**SUGGESTED AUDIENCE**
6th – 10th grade students

**SUBJECT & INDUSTRY LINKS**

**Applicable Subjects**
- mathematics
- art
- engineering

**Applicable Industries**
- engineering & product design
- research & education
- model making & entertainment
## LESSON OVERVIEW

<table>
<thead>
<tr>
<th>Time</th>
<th>Activity</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 min</td>
<td><strong>Do Now</strong></td>
<td>Students will attempt to translate their view of a 3D printed dodecahedron into a paper sketch.</td>
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<tr>
<td>10 min</td>
<td><strong>Foundation</strong></td>
<td>Introduce students to the “rationalization of space” and its connection to many industries, including 3D modeling, animation, architecture, game design, fine arts, and engineering.</td>
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</tbody>
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| 30 min| **Exploration**   | **Projections** Using a 3D printed Dodecahedron Kit, guide students through drawing a more precise dodecahedron. Use the “projection box” to teach principles of orthographic views as a way of translating three-dimensional space.  
**Golden Ratio** Walk students through an exploration of the proportions (the Golden Ratio) inherent in a dodecahedron and their link to orthographic views. Lead students in the construction of the Golden Ratio and guide them to create one face of a dodecahedron (a pentagon).  
**Perspective Drawing** Explain the difference between orthographic projections and perspective drawings. Teach students how to use the Dodecahedron Kit to create a perspective drawing of a dodecahedron. |
| 10 min| **Retrospective** | **Connecting the Dots** The class will share their creations and discuss other approaches for developing orthographic views of other objects. |
| 5 min | **Closing**       | **Looking Forward** Discuss the relevance of 3D concepts explored in this lesson, in understanding our world and in sparking innovations across many industries. |

* Download the .STL and .FORM files at formlabs.com/lesson-plan-dodecahedron
ACTIVITY ONE

5 min  Do Now

1. Distribute one 3D printed dodecahedron to each student (or student group).

2. Lead an interactive group discussion around the dodecahedron geometry.

3. Explain that the process of drawing objects is translating three-dimensional space into two-dimensional space.

4. Invite students to draw a dodecahedron, using the 3D printed model as a reference.

   Tips: Be encouraging, this is a challenging task. Considering hinting that you will be teaching them ‘tricks’ for sketching 3D shapes.

5. Give examples of why being able to translate three-dimensional geometries into two dimensions is an important skill.

Dodecahedron Geometry and History
Dodecahedrons are one of five Platonic solids identified 2,400 years ago by the Greek philosopher Plato. They are characterized by have same-length edges and same-shape faces, and are building blocks of our three-dimensional world. Dodecahedrons have 20 equal-length edges and 12 pentagonal faces. Other Platonic solids: cube, tetrahedron, octahedron, icosahedron.

Other Paths to Explore: Plato, Platonic Solids, Archimedean Solids.
ACTIVITY TWO

10 min  Foundation

1. Review students’ sketches and point out that the many edges and faces of a dodecahedron can make the shape confusing to draw.

2. Describe that Pre-Renaissance artists also had difficulty translating three-dimensions into two-dimensional space. Show and discuss Lorenzetti painting.

3. Show students two artists’ (da Vinci and Durer) drawings of dodecahedrons.

4. Discuss the inter-relatedness of geometry, natural science, and art using the collaboration between Pacioli and da Vinci as an example.

5. Introduce the concept of “rationalization of space” and “projective geometry” in the context of Durer’s “perspective machine.”

Renaissance Impact
The Renaissance (1300 - 1600) marked a transitional period for the understanding of how to represent 3D objects in 2D. Ambrogio Lorenzetti’s painting portrays an unrealistic perspective. Renaissance artists and mathematicians, however, worked together to understand and depict three-dimensional space. These artists included Leonardo da Vinci, who prepared drawings of Platonic solids for the mathematician Luca Pacioli. Similarly, Albrecht Durer was fascinated by the connection between geometry, natural science, and art. Artists like Durer discovered that 3D space can be “projected” onto 2D surfaces in a predictable way. His image of artists using a “perspective machine” illustrates one way to “rationalize space.” This was the origin of projective geometry, which is used in many fields of STEAM to this day.

Other Paths to Explore:  Lorenzetti, Pacioli, da Vinci, Durer.
**ACTIVITY THREE**

30 min  **Exploration**

1. Help students set up their Kits, with the dodecahedron in the middle and with an acetate sheet ("Picture Plane") taped to a side of the cube ("Projection Box") that is perpendicular to the table. Secure the Projection Box to the table with tape.

2. Instruct students to hold a permanent marker in their dominant hand, and the Eyepoint Stick in their other hand.

3. Have students look at the dodecahedron at eye level, while holding the Eyepoint Stick vertically and halfway between their eyes and the Picture Plane.

4. Instruct students to close one eye, align the top of the Eyepoint Stick with a vertex of the dodecahedron, and to carefully trace the dodecahedron on the Picture Plane.

**Projective Geometry** is the process of translating a three-dimensional object onto a two-dimensional plane.

**Orthographic Views** are one way that projective geometry is used. The view, or “projection” of the dodecahedron differs depending on the viewing angle. With the dodecahedron inside the Projection Box, there are three distinct **parallel views** - one per dimension. This explains why it takes three **orthographic views** to describe a three-dimensional object.

**Linear Perspective** is another way that projective geometry is used. This method uses a conical projection, rather than a parallel projection for a 3D-to-2D translation. **Linear perspective** helps artists mimic what the human eye sees.

Renaissance artists used both methods as tools for depicting three-dimensional space in ways that had never been possible before.
ACTIVITY THREE – CONTINUED

30 min  **Exploration**

1. Review students’ conical projection drawings.

2. Explain to students that complete orthographic views should make it possible to create the object in physical form. Ask students how they would approach creating accurate orthographic views.

3. Explore the relationship between the Fibonacci Sequence and the Golden Ratio with students.

4. Ask students to group their Measuring Sticks by length. There are two small (s), three medium (m), and two large (l) sticks.

5. Use the Measuring Sticks to bring the Fibonacci Sequence and Golden Ratio to life.
   
   \[ s + s = m \]
   
   is analogous to
   
   \[ 1 + 1 = 2 \]

6. Ask students what the next stick length in the sequence would be.
   
   \[ s + m = l \]

7. Challenge students to find the Golden Ratio proportions in the dodecahedron.
   
   \[ s = \text{dodecahedron edge} \]
   
   \[ m = \text{pentagonal face diagonal} \]
   
   \[ l = \text{distance between pentagon centers} \]

8. Point out some real-world examples and applications of the Golden Ratio.

**Golden Ratio** The geometry of pentagons and dodecahedrons is linked to the Golden Ratio, a phenomena that Leonardo’s friend, Luca Pacioli, wrote a whole book about. For centuries, artists, mathematicians, and scientists have found this number very interesting, due to its abundant natural presence.

**Fibonacci Sequence** The Golden Ratio is related to a special sequence of numbers, whereby you start with two numbers and each subsequent number equals the sum of the two previous numbers.

   e.g. 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89...

As the sequence progresses, the result of dividing one number in the sequence by the previous (e.g. 89/55) converges on an irrational number, \( \phi = 1.618... \), which is called the golden ratio.

**Self-Similarity** is a key property of the Golden Ratio. Numbers within the Fibonacci Sequence have this proportional relationship in common.
ACTIVITY THREE – CONTINUED

30 min  Exploration

1. Challenge students to make two different golden rectangles using the Measuring Sticks. Review relationships from last exercise.

2. Walk students through constructing a golden rectangle using a compass and ruler.
   a. Set compass to radius ‘r’
   b. Draw circle with radius ‘r’
   c. Use ruler to draw square in top right circle quadrant
   d. Use ruler to find the midpoint on the bottom side of the square. Use ruler to connect this midpoint to the top right corner of the square.
   e. Set compass to distance of last drawn line. Draw a new circle, with the origin at the previous midpoint from step ‘d.’
   f. Extend bottom side of square to right side of new circle. Then draw the two missing sides of the Golden Rectangle.

Other Paths to Explore: Using compass and ruler to draw a pentagon.
ACTIVITY THREE – CONTINUED

30 min  Exploration

1. Present two orthographic views of the dodecahedron to the students (Side and Top in this example).

2. Explain how two orthographic views can be used to generate a linear perspective of a dodecahedron.

Orthographic View Explained
The “Picture Plane Projection” shows the intersection of two views (Top and Side) of the same dodecahedron. The “Picture Plane” receives the conical projection. In this case, there are two picture planes: one from a Side view and one from a Front view. In each of the two views, the points of the dodecahedron are translated onto the Picture Plane Projection. It is the intersection of these views that illuminates a 2D depiction, the “linear perspective drawing,” of the 3D dodecahedron.
ACTIVITY FOUR

10 min  **Retrospective**

1. Review the pathway students followed in understanding what a dodecahedron is and how the Projection Box enabled them to create orthographic views, which then aided in creating a linear perspective drawing.

2. Engage with students about the difficulties around creating a perspective view of **more complicated objects** and landscapes.

3. Discuss ways 3D data are manipulated today: computer-aided-design (CAD), 3D scanning, photogrammetry-based model creation, for instance. Computer and software advancements have greatly enhanced the back-and-forth translation between two-dimensions and three-dimensions.

4. Highlight the next step that 3D printing enables, to bring three-dimensional models to life!

**Fun Fact** Just as 3D printing software creates 2D stacked slides of 3D parts, 500 years ago, Albrecht Durer also explored this ‘slicing’ concept as a way of creating accurate perspective drawings of humans.
ACTIVITY FIVE

5 min  CLOSING

1. Lead a brainstorming session with the students, around careers that may require working knowledge of 3D space.

2. Build on students’ ideas, by making links to classes that relate to those careers. Some example courses: art, computer science, engineering, geometry, woodworking, machine shop, photography, game design, and more.

We hope this lesson plan was a helpful look at a classroom application for desktop SLA! If you’re interested in bringing the plan to your classroom, download the .STL file for the Dodecahedron Kit.

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